

$\mathcal{N} = (0, 2)$ Deformation of the $\mathcal{N} = (2, 2)$ Wess-Zumino Model in Two Dimensions

M. Shifman^a and A. Yung^{a,b}

^a*William I. Fine Theoretical Physics Institute, University of Minnesota,
Minneapolis, MN 55455, USA*

^b*Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia*

Abstract

We construct a simple $\mathcal{N} = (0, 2)$ deformation of the two-dimensional Wess–Zumino model. In addition to superpotential, it includes a “twisted” superpotential. Supersymmetry may or may not be spontaneously broken at the classical level. In the latter case an extra right-handed fermion field ζ_R involved in the $\mathcal{N} = (0, 2)$ deformation plays the role of Goldstino.

Recently it was found [1, 2] that non-Abelian string solitons in certain $\mathcal{N} = 1$ bulk gauge theories are described on the world sheet by $\mathcal{N} = (0, 2)$ deformations of the $\text{CP}(N - 1)$ models. This finding raised interest to $\mathcal{N} = (0, 2)$ deformations of two-dimensional $\mathcal{N} = (2, 2)$ models in general. Here we will consider $\mathcal{N} = (0, 2)$ deformations of the Wess–Zumino model [3]. General elements of $\mathcal{N} = (0, 2)$ deformations were worked out by Witten [4, 5]. A broad class of the $(0, 2)$ Landau–Ginzburg models were analyzed, from various perspectives, in [6, 7, 8]. The prime interest of these studies was the flow of the $(0, 2)$ Landau–Ginzburg models to non-trivial $(0, 2)$ superconformal field theories [6, 7], and $\mathcal{N} = (0, 2)$ analogs of the topological rings in the $\mathcal{N} = (2, 2)$ theories [8].

Here we will consider the $\mathcal{N} = (0, 2)$ deformation of the Wess–Zumino model with the emphasis on an aspect which will be thoroughly studied in a subsequent publication [9], namely, spontaneous breaking/nonbreaking of supersymmetry. Related issues of interest are (i) a nonrenormalization theorem; (ii) BPS saturation of possible kinks. We use a formalism which is simple enough and is adequate to the problem. It parallelizes the formalism exploited in [2] to construct the heterotic $\text{CP}(N - 1)$ model, see also [10]. For simplicity we consider only the simplest version of the Wess–Zumino model, in which interactions come only from the potential term. Generalizations are straightforward, see also [6, 7].

We find that, even though the $\mathcal{N} = (0, 2)$ supersymmetry is implemented at the Lagrangian level, generically supersymmetry is spontaneously broken at the tree level. One can fine-tune a free parameter of the model in such a way that it stays unbroken at the tree level, and then (presumably) to any finite order in perturbation theory.

Two space-time coordinates are

$$x^\mu = \{t, z\}, \quad \mu = 0, 1. \quad (1)$$

The $\mathcal{N} = (2, 2)$ superspace is spanned by¹

$$\{x^\mu, \theta_\alpha, \bar{\theta}_\beta\}, \quad \alpha, \beta = 1, 2. \quad (2)$$

In addition to the standard chiral superfields Φ^a of the conventional Wess–Zumino model, we will introduce $\mathcal{N} = (0, 2)$ superfields

$$\begin{aligned} \mathcal{B} &= \left\{ \zeta_R(x^\mu + i\bar{\theta}\gamma^\mu\theta) + \sqrt{2}\theta_R\mathcal{F} \right\} \theta_L^\dagger, \\ \mathcal{B}^\dagger &= \theta_L \left\{ \zeta_R^\dagger(x^\mu - i\bar{\theta}\gamma^\mu\theta) + \sqrt{2}\theta_R^\dagger\mathcal{F}^\dagger \right\}. \end{aligned} \quad (3)$$

¹The gamma-matrices are chosen as $\gamma^0 = \gamma^t = \sigma_2$, $\gamma^1 = \gamma^z = i\sigma_1$, $\gamma_5 \equiv \gamma^0\gamma^1 = \sigma_3$. Moreover, $\bar{\theta} = \theta^\dagger\gamma^0$. With these definitions, the $\alpha = 1$ spinor component is right-handed while $\alpha = 2$ left-handed.

Since θ_L and θ_L^\dagger enter in Eq. (3) explicitly, \mathcal{B} and \mathcal{B}^\dagger are *not* superfields with regards to the supertransformations with parameters $\epsilon_L, \epsilon_L^\dagger$. These supertransformations are absent in the heterotic model. Only those survive which are associated with $\epsilon_R, \epsilon_R^\dagger$. Note that \mathcal{B} and \mathcal{B}^\dagger are superfields with regards to the shifts with $\epsilon_R, \epsilon_R^\dagger$. As usual, we will introduce a shorthand for the chiral coordinate

$$\tilde{x}^\mu = x^\mu + i\bar{\theta}\gamma^\mu\theta. \quad (4)$$

Then the transformation laws with the parameters $\epsilon_R, \epsilon_R^\dagger$ are as follows (we set $\epsilon_L = \epsilon_L^\dagger = 0$):

$$\delta\theta_R = \epsilon_R, \quad \delta\theta_R^\dagger = \epsilon_R^\dagger, \quad \delta\tilde{x}^0 = 2i\epsilon_R^\dagger\theta_R, \quad \delta\tilde{x}^1 = 2i\epsilon_R^\dagger\theta_R. \quad (5)$$

With respect to such supertransformations, \mathcal{B} and \mathcal{B}^\dagger are superfields. Indeed,

$$\delta\zeta_R = \sqrt{2}\mathcal{F}\epsilon_R, \quad \delta\mathcal{F} = \sqrt{2}i(\partial_L\zeta_R)\epsilon_R^\dagger, \quad (6)$$

plus Hermitian conjugate transformations.

Thus, the boson sector of the deformed model coincides with that of the conventional Wess–Zumino model, while the fermion sector is expanded. In addition to the fermion fields $\psi_{R,L}^a$ of the Wess–Zumino model it includes a right-handed fermion field ζ_R .

The $\mathcal{N} = (0, 2)$ action can be written as

$$\begin{aligned} S &= \int d^2x \left\{ d^4\theta \Phi^{a\dagger}\Phi^a + [d^2\theta \mathcal{W}(\Phi^a) + \text{H.c.}] + \Delta\mathcal{L}_h \right\}, \\ \Delta\mathcal{L}_h &= \left\{ \sqrt{2}\kappa \int d\theta_L^\dagger d\theta_R \mathcal{B} + \text{H.c.} \right\} \\ &\quad - 2 \int d^4\theta \mathcal{B}^\dagger\mathcal{B} + 2 \left\{ \int d\theta_L^\dagger d\theta_R \theta_L d\theta_L \mathcal{B} \mathcal{S}(\Phi^a) + \text{H.c.} \right\}, \end{aligned} \quad (7)$$

where the second term presents the heterotic deformation, \mathcal{W} is the superpotential, while $\mathcal{S}(\Phi^a)$ is a function of the chiral superfield Φ which couples the heterotic sector to the conventional Wess–Zumino model. (Some generalizations will be considered later.) Let us call it h -superpotential. Moreover, κ is a constant of dimension of mass. Note that both, superpotential and h -superpotential have dimensions of mass too. The terms containing \mathcal{B} and given by integrals over a reduced superspace will be referred to as h terms. Adding a constant to the h -superpotential is equivalent to

shifting κ since they enter only in the combination $\mathcal{S}(\phi^a) + \kappa$, see Eq. (9). One can use this freedom to fix the value of \mathcal{S} at some given point, without loss of generality.

In components

$$\begin{aligned} \mathcal{L} = & \partial_\mu \phi^{a\dagger} \partial^\mu \phi^a + \bar{\psi}^a \gamma^\mu i \partial_\mu \psi^a + F^{a\dagger} F^a \\ & + \{ F^a \partial_a \mathcal{W} - (\partial_a \partial_b \mathcal{W}) (\psi_L^a \psi_R^b) + \text{H.c.} \} + \Delta \mathcal{L}_h \end{aligned} \quad (8)$$

where

$$\Delta \mathcal{L}_h = \zeta_R^\dagger i \partial_L \zeta_R + \mathcal{F}^\dagger \mathcal{F} + \{ \mathcal{F} [\mathcal{S}(\phi^a) + \kappa] - \zeta_R \psi_L^a \partial_a \mathcal{S} + \text{H.c.} \}, \quad (9)$$

and

$$\partial_L = \partial_t + \partial_z, \quad \partial_R = \partial_t - \partial_z. \quad (10)$$

The auxiliary superfield \mathcal{F} , as usual, can be eliminated via equations of motion,

$$\mathcal{F}^\dagger = -[\mathcal{S}(\phi^a) + \kappa]. \quad (11)$$

Then the bosonic part of \mathcal{L}_h takes the form

$$\mathcal{L}_{h,\text{bos}} = -|\mathcal{S}(\phi^a) + \kappa|^2. \quad (12)$$

Adding the Wess–Zumino part we obtain the scalar potential,

$$V = |\partial_a \mathcal{W}|^2 + |\mathcal{S}(\phi^a) + \kappa|^2. \quad (13)$$

Supersymmetric vacua exist (at the classical level) provided that the set of equations

$$\partial_a \mathcal{W} = 0 \quad (\text{all } a), \quad \mathcal{S}(\phi^a) + \kappa = 0 \quad (14)$$

are satisfied at one or more critical points ϕ_* . Considering κ as a free parameter one can always fine-tune it in such a way that at least one vacuum (a solution $\partial_a \mathcal{W}(\phi_*) = 0$) will be classically supersymmetric.

It is instructive to derive conserved supercurrents. If in the undistorted model with $\mathcal{N} = (2, 2)$ supersymmetry we had four conserved supercurrents, J_L^μ , J_R^μ and their complex conjugated, now we expect only two of those to survive. The conserved components are

$$\begin{aligned} J_L^\mu &= \sqrt{2} \left\{ i \nu^\mu \psi_R^\dagger F + i \nu^\mu \zeta_R^\dagger \mathcal{F} + \bar{\nu}^\mu \psi_L \partial_L \phi^\dagger \right\} \\ (J_L^\mu)^\dagger &= \sqrt{2} \left\{ -i \nu^\mu \psi_R F^\dagger - i \nu^\mu \zeta_R \mathcal{F}^\dagger + \bar{\nu}^\mu \psi_L^\dagger \partial_L \phi \right\} \end{aligned} \quad (15)$$

where summation over a is implicit, and we defined two conjugated 2-vectors,

$$\nu^\mu = \{1, 1\}, \quad \bar{\nu}^\mu = \{1, -1\}. \quad (16)$$

The corresponding superalgebra is as follows:

$$\{Q_L^\dagger, Q_L\} = 2(H - P^z). \quad (17)$$

From Eq. (17) it is obvious that massless right-movers can (and do) form short (single-state) “multiplets.”

In the $\mathcal{N} = (2, 2)$ Wess–Zumino model ($\mathcal{B} = 0$) there is a relation for the dilatation operator

$$(\gamma_\mu J^\mu)_{L,R} = i 2\sqrt{2} F (\psi^\dagger)_{L,R}. \quad (18)$$

In the $\mathcal{N} = (0, 2)$ -deformed model the analog of this relation is

$$\bar{\nu}_\mu J_L^\mu = i 2\sqrt{2} \left(F \psi_R^\dagger + \mathcal{F} \zeta_R^\dagger \right). \quad (19)$$

Some generalizations

In addition to (9), one can couple the \mathcal{B} field to other fields through a number of extra terms, for instance,

$$\int d^4\theta \mathcal{B}^\dagger \mathcal{B} f(\Phi \Phi^\dagger) \text{ or } \int d^4\theta \mathcal{B} \tilde{f}(\Phi \Phi^\dagger) + \text{H.c.} \quad (20)$$

The first term gives, in particular, a coupling of the ζ kinetic term with the ϕ, ϕ^\dagger fields. As was mentioned, such interactions will not be considered for the time being. The second term was considered in [2]. It generates the $\zeta_R \partial_L \phi^\dagger \psi_R$ interaction and an additional bifermion term $\psi_L^\dagger \psi_R$ in (11), as well as $\zeta_R^\dagger \psi_L$ in F , resulting in four-fermion interactions in the Lagrangian.

Nonrenormalization of h terms

As well-known, F terms in the effective Lagrangian in the $\mathcal{N} = (2, 2)$ theory are protected from renormalizations by nonrenormalization theorems [11, 12]. Thus, the superpotential term, being an integral over a reduced superspace, is unaffected by loops. Since the h -terms are also given by integrals over a reduced superspace, similar theorems can be established in the $\mathcal{N} = (0, 2)$ models for these terms. An appropriate choice of the background field in this case is

$$\begin{aligned} \Phi_b^\dagger &= 0, & \Phi_b &= C_1 + C_2^\alpha \theta_\alpha + C_3 \theta^2, \\ \mathcal{B}_b^\dagger &= 0, & \mathcal{B}_b &= C_4 \theta_L^\dagger + C_5 \theta_R \theta_L^\dagger, \end{aligned} \quad (21)$$

where the subscript b mark the background fields, and $C_{1,2,3,4,5}$ are *c*-numerical *constants*. This choice assumes that Φ and Φ^\dagger are treated as independent variables, not connected by complex conjugation, and so are \mathcal{B} and \mathcal{B}^\dagger (i.e. we keep in mind a kind of analytic continuation). The x independent fields (21) are invariant under the action of Q_L^\dagger . Next, to do the calculation of the effective action we decompose the superfields

$$\begin{aligned}\Phi &= \Phi_b + \Phi_{\text{qu}}, & \Phi^\dagger &= \Phi_b + \Phi_{\text{qu}}^\dagger, \\ \mathcal{B} &= \mathcal{B}_b + \Phi_{\text{qu}}, & \mathcal{B}^\dagger &= \mathcal{B}_b + \mathcal{B}_{\text{qu}}^\dagger,\end{aligned}\tag{22}$$

where the subscript qu denotes the quantum part of the superfield, expand the action in $\Phi_{\text{qu}}, \mathcal{B}_{\text{qu}}$ dropping the linear terms, and treat the remainder as the action for the quantum fields. We, then, integrate the quantum fields over, order by order, keeping the background field fixed. The crucial point is that in the given background field (a) the terms containing \mathcal{W} and \mathcal{B} , *without* \mathcal{W}^\dagger and \mathcal{B}^\dagger , do *not* vanish, and (b) there exists an exact supersymmetry under Q_L^\dagger -generated supertransformations.

After substituting in loops Green's functions in the given background and integrating over all vertices except the first one we arrive at an expression of the type

$$\int d\theta_R^\dagger \times \left(\text{a } \theta_R^\dagger \text{ independent function} \right) = 0.\tag{23}$$

The θ_R^\dagger independence follows from the fact that our superspace is homogeneous in the θ_R^\dagger direction even in the presence of the background field (21). This completes the proof of nonrenormalization of F and h terms in the $\mathcal{N} = (0, 2)$ theory.

A subtle point here is that this proof tacitly assumes the absence of infrared singularities. Thus, it is certainly valid for the Wilsonian effective action [13]. If infrared contributions are included (i.e. the generator of 1-particle irreducible amplitudes is studied) the question should be investigated on a case by case basis.

Kinks

$\mathcal{N} = (2, 2)$ models with two or more supersymmetric vacua (two or more zeros of $\partial_a \mathcal{W}$) support interpolating kink solutions which, typically, are 1/2 BPS saturated, i.e. preserve two out of four supersymmetries of the $\mathcal{N} = (2, 2)$ model under consideration. Adding an $\mathcal{N} = (0, 2)$ deformation can destroy vacuum degeneracy and, thus, eliminate kinks altogether. Even if we choose a deformation of a special form which does not break $\mathcal{N} = (0, 2)$ supersymmetry in two or more vacua, BPS-saturated kinks do not exist in such theories. This is readily seen through the Bogomoln'yi completion [14]. With the heterotic deformation switched on the Bogomol'nyi completion of the bosonic part of the energy functional is impossible, generally speaking.

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